An iterative AC-SCOPF approach managing the contingency and corrective control failure uncertainties with a probabilistic guarantee

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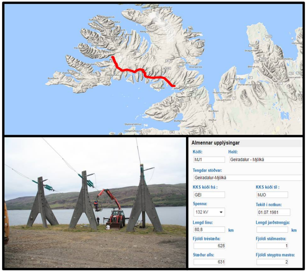
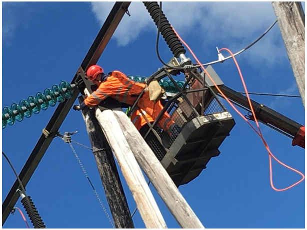
Presentation outline

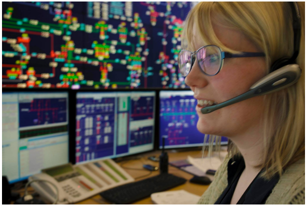
1. Background & motivation
2. Problem formulation
3. Algorithmic solution approach
4. Case study results
5. Wrap-up & future work

Reliability management

- Making decisions under uncertainty, from long-term system development to real-time system operation.







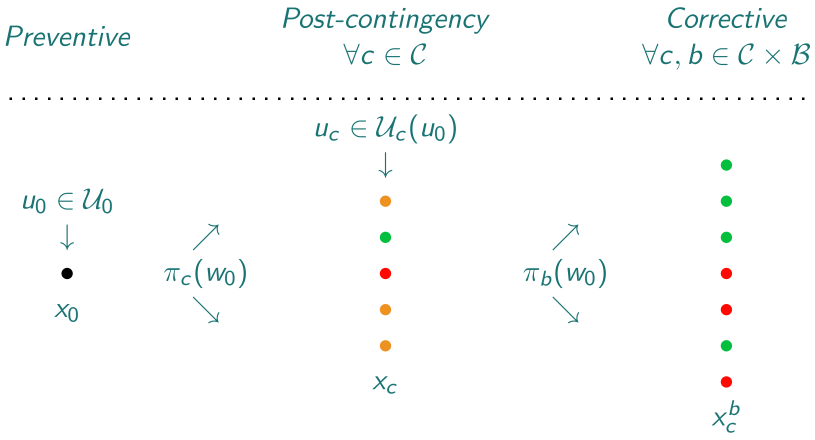
- A reliability criterion sets the basis to determine whether or not the system reliability is acceptable.

Real-time operation reliability management

Horizon: (5' $\sim$ 15')

- Power injections assumed relatively predictable.
- Uncertainty on:
  - $\rightarrow$ occurrence of contingencies $c \in \mathcal{C}$;
  - $\rightarrow$ behavior of post-contingency corrective controls $b \in \mathcal{B}$.
- Decisions to:
  - $\rightarrow$ apply preventive (pre-contingency) control $u_0 \in \mathcal{U}_0(x_0)$?
  - $\rightarrow$ prepare post-contingency corrective controls $u_c \in \mathcal{U}_c\,(u_0)\,\forall c \in \mathcal{C}$?

Transitions of the system state



w_0: spatial/temporal correlation in transition probabilites.

Security Constrained Optimal Power Flow

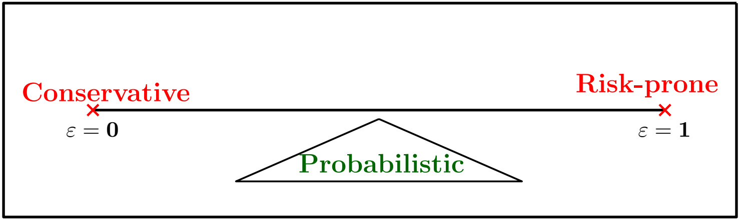
The N-1 approach

- Maintain stable equilibrium (system operational limits) following any single outage,
- but, how to rely on uncertain corrective control?
  - → don't, since it may fail (conservative)?
  - → do, just neglect failure (risk-prone)?

Security Constrained Optimal Power Flow

The probabilistic approach

- Maintain stable equilibrium (system operational limits), at least with a certain confidence,
  - $\rightarrow$ so that the joint probability of violating operational limits remains below a tolerance $\varepsilon \in [0, 1]$.



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Steady-state operational limits

- AC power flow (rectangular coordinates);
- voltage magnitude bounds per node;
- voltage angle difference & apparent power flow bounds per branch;
  - → less restrictive for the intermediate problem stage;
- active & reactive power generation bounds per unit;
  - → ramping restrictions between preventive & corrective active power dispatch;
- voltage set-points per unit;
- no loss of load.

State-of-the-art determinstic-constrained problem

$$\min_{\mathbf{u}\in\mathbf{U}} CP(x_0, u_0) \tag{1}$$

$$h_0(x_0, u_0) \leq 0; \tag{2}$$

$$h_c^s(x_c, u_0) \leq 0 \quad \forall c \in \mathcal{C}; \tag{3}$$

$$h_c(x_c^w, u_c) \leq 0 \quad \forall c \in \mathcal{C}; \tag{4}$$

$$\mathbf{u} \in \mathbf{U} \equiv \{u_0 \in \mathcal{U}_0(x_0); u_c \in \mathcal{U}_c(x_0, u_0, c) \forall c \in \mathcal{C}\}. \tag{5}$$

- Minimizing the cost of preventive operation (**??**);
- $h_.(x_., u_.)$ groups all operational limits for preventive, intermediate and corrective stage (**??** - **??**);
- only for perfectly working corrective controls (x_c^w);
- preventive & corrective decisions are coupled (**??**).

Our chance-constrained problem

$$\min_{\mathbf{u}\in\mathbf{U}} CP(x_0, u_0) + \sum_{c\in\mathcal{C}} \pi_c \cdot CC(x_0, u_0, c, u_c); \tag{6}$$

$$h_0(x_0, u_0) \leq 0; \tag{7}$$

$$\mathbb{P}\left\{ \begin{array}{l} h_c^s(x_c, u_0) \leq 0 \\ h_c(x_c^b, u_c) \leq 0 \end{array} \middle| (c, b) \in \mathcal{C} \times \mathcal{B} \right\} \geq 1 - \varepsilon; \tag{8}$$

$$\mathbf{u} \in \mathbf{U} \equiv \{u_0 \in \mathcal{U}_0(x_0); u_c \in \mathcal{U}_c(x_0, u_0, c) \forall c \in \mathcal{C}\}. \tag{9}$$

- Also includes expectation of corrective stage costs (**??**);
- post-contingency operational limits to hold at least with $(1 - \varepsilon)$ probability (**??**);
- taking into account contingency occurrence probabilities & corrective control behavior probabilities.

Chance-constraint reformulation – step one

$$\mathbb{P}\left\{ \begin{array}{l} h_c^s(x_c, u_0) \leq 0 \\ h_c(x_c^b, u_c) \leq 0 \end{array} \middle| (c, b) \in \mathcal{C} \times \mathcal{B} \right\} \geq 1 - \varepsilon;$$

- LH-side measures the probability of all post-contingency stages meeting operational constraints;

Chance-constraint reformulation – step one

$$\mathbb{P}\left\{ \begin{array}{l} h_c^s(x_c, u_0) \leq 0 \\ h_c(x_c^b, u_c) \leq 0 \end{array} \middle| (c, b) \in \mathcal{C} \times \mathcal{B} \right\} \geq 1 - \varepsilon;$$

- LH-side measures the probability of all post-contingency stages meeting operational constraints;
- introducing indicator function $\mathcal{I}(x_0, u_0, c, u_c, b)$ to show post-contingency constraint violations,

$$\mathcal{I}(x_0, u_0, c, u_c, b) = \begin{cases} 1 \equiv \{h_c^s(x_c, u_0) \not\leq 0 \vee h_c(x_c^b, u_c) \not\leq 0\} \\ 0 \equiv \{h_c^s(x_c, u_0) \leq 0 \wedge h_c(x_c^b, u_c) \leq 0\} \end{cases}.$$

Chance-constraint reformulation – step one

$$\mathbb{P}\left\{ \begin{array}{l} h_c^s(x_c, u_0) \leq 0 \\ h_c(x_c^b, u_c) \leq 0 \end{array} \middle| (c, b) \in \mathcal{C} \times \mathcal{B} \right\} \geq 1 - \varepsilon;$$

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- when sets of contingencies and corrective control behaviors are discrete, one may re-write the chance-constraint as,

$$1 - \sum_{c \in \mathcal{C}} \pi_c \sum_{b \in \mathcal{B}} \pi_c^b \cdot \mathcal{I}(x_0, u_0, c, u_c, b) \geq 1 - \varepsilon.$$

Our chance-constrained problem

$$\min_{\mathbf{u}\in\mathbf{U}} CP(x_0, u_0) + \sum_{c\in\mathcal{C}} \pi_c \cdot CC(x_0, u_0, c, u_c); \quad (10)$$

$$h_0(x_0, u_0) \leq 0; \quad (11)$$

$$1 - \sum_{c\in\mathcal{C}} \pi_c \sum_{b\in\mathcal{B}} \pi_c^b \cdot \mathcal{I}(x_0, u_0, c, u_c, b) \geq 1 - \varepsilon; \quad (12)$$

$$\mathbf{u} \in \mathbf{U} \equiv \{u_0 \in \mathcal{U}_0(x_0); u_c \in \mathcal{U}_c(x_0, u_0, c) \forall c \in \mathcal{C}\}. \quad (13)$$

- Reformulated chance-constraint (**??**) sums the indicator function over all contingencies & corrective control behaviors!

Chance-constraint reformulation – step two

$$\mathbb{P}\left\{ \begin{array}{l} h_c^s(x_c, u_0) \leq 0 \\ h_c(x_c^b, u_c) \leq 0 \end{array} \middle| (c, b) \in \mathcal{C} \times \mathcal{B} \right\} \geq 1 - \varepsilon;$$

- Intermediate stage constraints need to hold to keep the system functional;
- we can partly replace indicator function $\mathcal{I}(x_0, u_0, c, u_c, b)$ with auxiliary binary variables ($p_c \in [0; 1]$) relaxing these constraints,

$$h_c^s(x_c, u_0) \leq p_c \cdot M, \forall c \in \mathcal{C}, \tag{14}$$

with M being a sufficiently large constant.

Chance-constraint reformulation – step two

$$\mathbb{P}\left\{\begin{array}{l} h^s_c(x_c, u_0) \leq 0 \\ h_c(x^b_c, u_c) \leq 0 \end{array} \middle| (c, b) \in \mathcal{C} \times \mathcal{B}\right\} \geq 1 - \varepsilon;$$

- In a cost minimization context, corrective control would only be selected when needed;
- that is, to alleviate some post-contingency constraint violation;
- when corrective control doesn't work, we'd have constraint violations;
- hence, every post-contingency stage with corrective actions contributes to the sum appearing in the LH-side of the chance constraint (**??**).

Chance-constraint reformulation – step two

- Every post-contingency stage with corrective actions contributes to the sum appearing in the LH-side of the chance constraint (**??**);
- we can partly replace indicator function $\mathcal{I}(x_0, u_0, c, u_c, b)$ with auxiliary binary variables ($i_c \in [0;1]$) showing the use of post-contingency corrective controls;

$$\mathcal{I}(x_0, u_0, c, u_c, b) \equiv \left\{ \begin{array}{l} h_c^s(x_c, u_0) \leq p_c \cdot M \\ h_c(x_c^w, u_c) \leq 0 \\ |u_0 - u_c| \leq i_c \cdot M \\ i_c \in [0;1] \end{array} \right\}, \forall c \in \mathcal{C}, \qquad (15)$$

with M being a sufficiently large constant.

Our chance-constrained problem

$$\min_{\mathbf{u}\in\mathbf{U}} CP(x_0,u_0) + \sum_{c\in\mathcal{C}} \pi_c \cdot CC(x_0,u_0,c,u_c); \tag{16}$$

$$h_0(x_0,u_0) \leq 0; \tag{17}$$

$$h_c^s(x_c,u_0) \leq p_c \cdot M, \forall c \in \mathcal{C}, \tag{18}$$

$$h_c(x_c^w,u_c) \leq 0, \forall c \in \mathcal{C}, \tag{19}$$

$$1 - \sum_{c\in\mathcal{C}} \pi_c \cdot [p_c + (1-\pi_c^w)\cdot i_c] \geq 1-\epsilon; \tag{20}$$

$$|u_0 - u_c| \leq i_c \cdot M, \forall c \in \mathcal{C}, \tag{21}$$

$$i_c + p_c \leq 1, \forall c \in \mathcal{C}, \tag{22}$$

$$i_c, p_c \in [0;1], \forall c \in \mathcal{C}, \tag{23}$$

$$\mathbf{u} \in \mathbf{U} \equiv \{u_0 \in \mathcal{U}_0(x_0); u_c \in \mathcal{U}_c(x_0,u_0,c) \forall c \in \mathcal{C}\}. \tag{24}$$

- Additional coupling constraints and binary vars (**??**,**??**-**??**).

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Algorithm motivation

The deterministic-constrained problem

- Optimistic attitude towards corrective control failures;
- a large-scale Mixed-Integer Non-Linear Programming (MINLP) problem;
- state-of-the-art solution approach is contingency filtering.

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- state-of-the-art solution approach is contingency filtering.

Our chance-constrained problem

- Reformulated as a MINLP;
- includes constraints from the optimistic version and then some more;
- how can we adapt contingency filtering schemes?

Solution principle

- Any chosen decision partitions the contingency set ...

Preventive Only $\mathcal{C}_{\mathcal{P}}$	Preventive & Corrective $\mathcal{C}_{\mathcal{C}}$	Not Secured $\mathcal{C}_{\mathcal{X}} = \mathcal{C} \setminus (\mathcal{C}_{\mathcal{C}} \cup \mathcal{C}_{\mathcal{P}})$

Solution principle

- Any chosen decision partitions the contingency set . . .

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+ we get a lower-bound for the probability of interest;

$$\mathbb{P}\Big\{ \dots \Big\} \geq 1 - \left(\sum_{c \in \mathcal{C}_{\mathcal{X}}} \pi_c + \sum_{c \in \mathcal{C}_{\mathcal{C}}} \pi_c \cdot \pi_c^f \right),$$

e.g., $\mathbb{P}\Big\{ \dots \Big\} \geq 1$ when all cntgcies are in preventive only.

Solution principle

- What if we grow secured contingency sub-sets $\mathcal{C}_{\mathcal{P}}, \mathcal{C}_{\mathcal{C}}$?

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- we could push the probability lower-bound upwards,

$$\mathbb{P}\Big\{ \dots \Big\} \geq 1 - \left(\sum_{c \in \mathcal{C}_{\mathcal{X}}} \pi_c + \sum_{c \in \mathcal{C}_{\mathcal{C}}} \pi_c \cdot \pi_c^f \right),$$

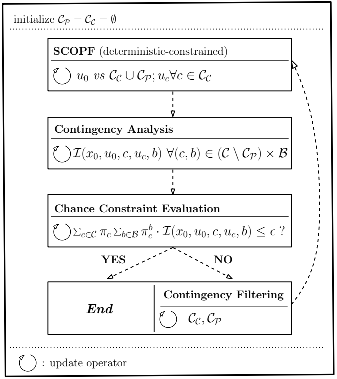
- until the actual probability value grows large enough.

Algorithmic decomposition overview

In a nutshell

1. update decisions *vs* deterministic constraints;
2. evaluate post-contingency violation probability;
3. update contingency subsets;
   - preventive only;
   - preventive & corrective;

– stop when reliability target is OK.



Algorithm components

Deterministic SCOPF

- standard IPOPT implementation *vs* given contingency subsets;

Contingency analysis

- examining both the working & failing behavior of corrective controls;
- per contingency & cc behavior, minimization of fictitious active/reactive power injections;
- returns a zero optimal value for feasible OPF instances;
- non-zero objective indicative of the magnitude of constraint violations implied by the contingency & cc behavior.

Contingency filtering variants

Probability-based (Pb)

- returns the most probable constraint-violating post-contingency stage (*e.g.*, immediately after contingency 14, after contingency 42 and cc failure, *etc.*).

Feasibility-based (Fb)

- returns the most severe constraint-violating post-contingency stage (*e.g.*, immediately after contingency 14, after contingency 42 and cc failure, *etc.*).

Risk-based (Rb)

- blends the former two, ranking post-contingency stages in probability $\times$ severity.

Contingency subset updating rule

- the goal is to push the lower bound on the constraint violation probability;
- we always tighten the security status of the filtered contingency:

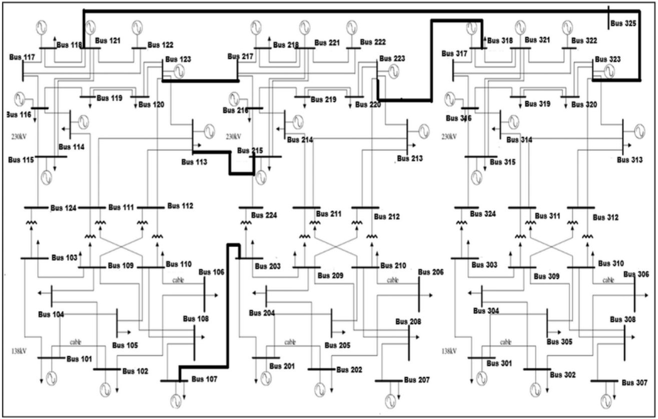
  $c \in \mathcal{C}_{\mathcal{X}} \rightarrow c \in \mathcal{C}_{\mathcal{C}}$: from not secured, to correctively & preventively secured;

  $c \in \mathcal{C}_{\mathcal{C}} \rightarrow c \in \mathcal{C}_{\mathcal{P}}$: from correctively & preventively secured, to preventively only secured;

- that is, make the contingency set partitioning more conservative.

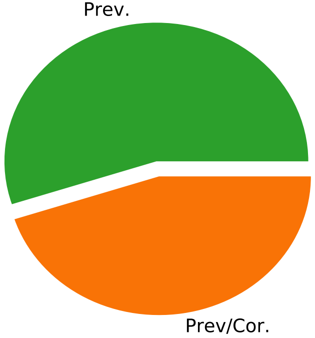
Presentation outline

The test-case



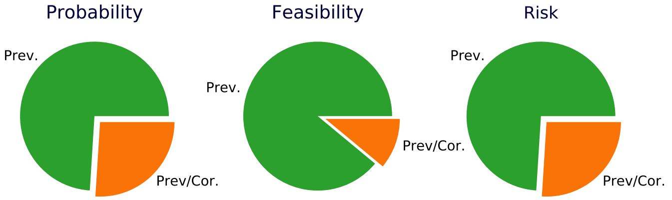
- 111 single component outages;
- Corrective control failure probability assumed 0.01.

Deterministic-constrained SCOPF



Total Cost ($)	881.62
Explicit Contingencies	4
Violation Probability	$1.91 \cdot 10^{-5}$ (*ex-post*)

Chance-constrained SCOPF ($\varepsilon = 10^{-5}$)



Filter	Probability	Feasibility	Risk
Total Cost ($)	892.37	896.78	892.37
Explicit Contingencies	13	5	7
Chance level	$9.85 \cdot 10^{-6}$	$5.28 \cdot 10^{-6}$	$9.85 \cdot 10^{-6}$

Chance-constrained SCOPF ($\varepsilon = 10^{-5}$)

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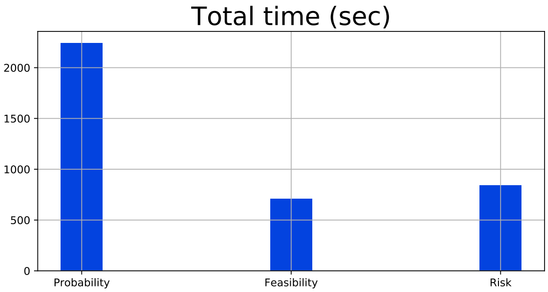
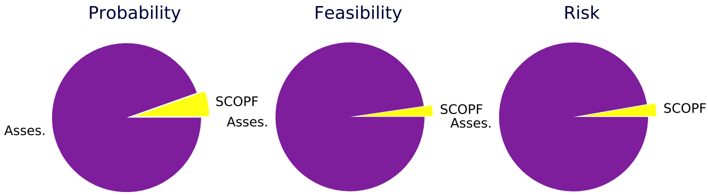
- More reliable solutions naturally more costly!
- risk-based filter returns the same solution & a sub-set of the cntgcies filtered by the probability-based;
- feasibility-based filter is more efficient (only 5 explicity cntgcies) yet more conservative (cost & chance-constraint level).

Chance-constrained SCOPF ($\varepsilon = 10^{-5}$)

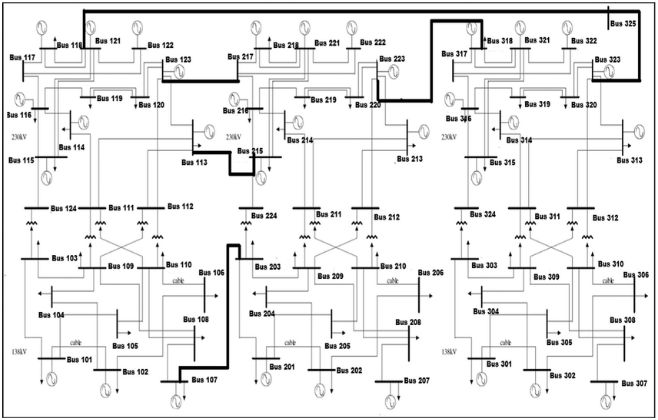




The story so far ...

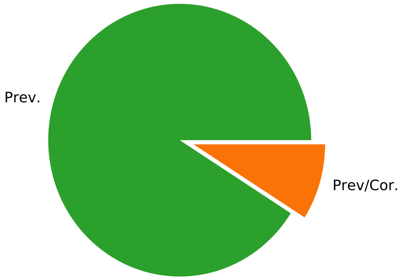
- The probability-based filter is outperformed:
  - → since all probabilities here are exogenous params, it carries no physical information;
  - → careful before generalizing to a different context (*e.g.* when cc failure probability depends on the chosen actions);
- the question of feasibility- *vs* risk-based filtering is open:
  - → feasibility-based is slightly more conservative;
  - → risk-based solves slightly slower;
- demonstrated results verified through sensitivity analysis (see full paper).

3 additional test-cases



- Taking into account weather impact on outage probabilities;
- assuming adverse weather hits any one of the system areas.

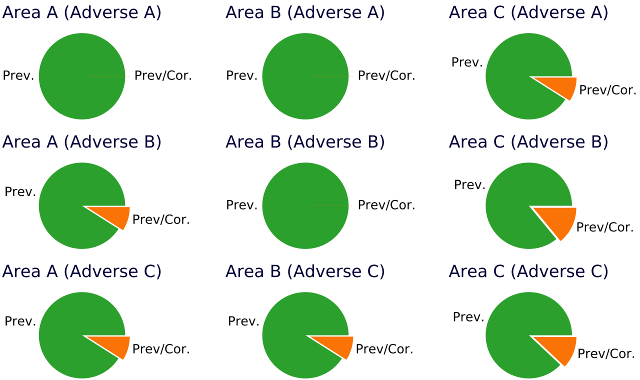
Feasibility-based filtering



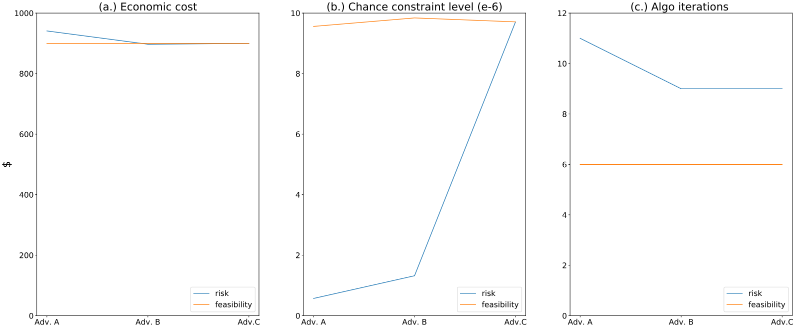
- Contingency filtering unaffected by occurrence probability.

Adverse Weather	Area A	Area B	Area C
Total Cost ($)		899.62	
Explicit Contingencies		6	
Violation Probability	$9.56 \cdot 10^{-6}$	$9.84 \cdot 10^{-6}$	$9.71 \cdot 10^{-6}$

Risk-based filtering



Risk-based filtering



- Feasibility-based filtering more robust w.r.t. the adverse weather.

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Wrap-up

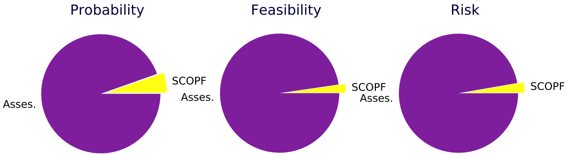
What?

- Practical algorithmic framework for chance-constrained mgmt of operational uncertainties in AC-SCOPF;

Why?

- Post-contingency corrective controls not 100% reliable;
- acknowledging threat explicitly & adopting a tolerance level;
- decision making problem is a slightly more complex variant of the classical AC-SCOPF;
- solution remains understandable & interpretable.

Future work on computational efficiency



- Assessment workload is the computational bottleneck;
- first opportunity to reduce computational times is parallellization;
- the more interesting is machine learning: predicting the objective of the single-contingency OPF problems.

Taking the story forward

How to apply this in practice?

- data collection & models;
- integration in operational practices as complex as any SCOPF variant;
- things are happening :)

How to apply this in other time-horizons?

- On-going work in planning *vs* power injection uncertainties.

Implementation details & full results

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Thanks for your attention

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